The Philosophy of Mathematics, Values and Keralese Mathematics

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Abstract

This paper explores the philosophical significance of the Keralese and Indian subcontinent contribution to history of mathematics. Identifying the most accurate genesis and trajectory of mathematical ideas in history that current knowledge allows should be the goal of every history of mathematics, and is consistent with any philosophy of mathematics. I argue for the need of a broader conceptualization of philosophy of than the traditional emphasis on scholastic enquiries into epistemology and ontology. For such an emphasis has been associated, though I add need not necessarily be so, with an ideological position that devalues non-European contributions to history of mathematics. The philosophy of mathematics needs to be broad enough to recognise the salient features of the discipline it reflects upon, namely mathematics.

Keywords: Non-european roots of mathematics; Keralese mathematics; Philosophy of mathematics; mathematics and values; history of mathematics.

1. What is the Business of the Philosophy of Mathematics?

Traditionally, in Western philosophy, mathematical knowledge has been understood as universal and absolute knowledge, whose epistemological status sets it above all other forms of knowledge. The traditional foundationalist schools of formalism, logicism and intuitionism sought to establish the absolute validity of mathematical knowledge by erecting foundational systems. Although modern philosophy of mathematics has in part moved away from this dogma of absolutism, it is still very influential, and needs to be critiqued. So I wish to begin by summarising some of the arguments against Absolutism, as this position has been termed (Ernest 1991, 1998).

My argument is that the claim of the absolute validity for mathematical knowledge cannot be sustained. The primary basis for this claim is that mathematical knowledge rests on certain and necessary proofs. But proof in mathematics assumes the truth, correctness, or consistency of an underlying axiom set, and of logical rules and axioms or postulates. The truth of this basis cannot

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be established on pain of creating a vicious circle (Lakatos 1962). Overall the correctness or consistency of mathematical theories and truths cannot be established in non-trivial cases (Gödel 1931).

Thus mathematical proof can be taken as absolutely correct only if certain unjustified assumptions made. First, it must be assumed that absolute standards of rigour are attained. But there are no grounds for assuming this (Tymoczko 1986). Second, it must be assumed that any proof can be made perfectly rigorous. But virtually all accepted mathematical proofs are informal proofs, and there are no grounds for assuming that such a transformation can be made (Lakatos 1978). Third, it must be assumed that the checking of rigorous proofs for correctness is possible. But checking is already deeply problematic, and the further formalizing of informal proofs will lengthen them and make checking practically impossible (MacKenzie 1993)

A final but inescapably telling argument will suffice to show that absolute rigour is an unattainable ideal. The argument is well-known. Mathematical proof as an epistemological warrant depends on the assumed safety of axiomatic systems and proof in mathematics. But Gödel's (1931) second incompleteness theorem means that consistency and hence establishing the correctness and safety of mathematical systems is indemonstrable. We can never be sure mathematics theories are safe, and hence we cannot claim their correctness, let alone their necessity or certainty. These arguments are necessarily compressed here, but are treated fully elsewhere (e.g., Ernest 1991, 1998). So the claim of absolute validity for mathematical knowledge is unjustified.

The past two decades has seen a growing acceptance of the weakness of absolutist accounts of mathematical knowledge and of the impossibility in establishing knowledge claims absolutely. In particular the 'maverick' tradition, to use Kitcher and Aspray's (1988) phrase, in the philosophy of mathematics questions the absolute status of mathematical knowledge and suggest that a reconceptualisation of philosophy of mathematics is needed (Davis and Hersh 1980, Lakatos 1976, Tymoczko 1986, Kitcher 1984, Ernest 1997). The main claim of the 'maverick' tradition is that mathematical knowledge is fallible. In addition, the narrow academic focus of the philosophy of mathematics on foundationist epistemology or on Platonistic ontology to the exclusion of the history and practice of mathematics, is viewed by many as misguided, and by some as damaging.

2. Reconceptualizing the Philosophy of Mathematics

Although a widespread goal of traditional philosophies of mathematics is to reconstruct mathematics in a vain foundationalist quest for certainty, but a number of philosophers of mathematics agree this goal is inappropriate. "To confuse description and programme - to confuse 'is' with 'ought to be' or 'should be' - is just as harmful in the philosophy of mathematics as elsewhere." (Körner 1960: 12), and "the job of the philosopher of mathematics is to describe and explain mathematics, not to reform it." (Maddy 1990: 28). Lakatos, in a characteristically witty and forceful way which paraphrases Kant indicates the direction that a reconceptualised philosophy of mathematics should follow. "The history of mathematics, lacking the guidance of philosophy has become blind, while the philosophy of mathematics turning its back on the...history of mathematics, has become empty" (1976: 2).

Building on these and other suggestions it might be expected that an adequate philosophy of mathematics should account for a number of aspects of mathematics including the following:

1. **Epistemology**: Mathematical knowledge; its character, genesis and justification, with special attention to the role of proof
2. **Theories**: Mathematical theories, both constructive and structural: their character and development, and issues of appraisal and evaluation
3. **Ontology**: The objects of mathematics: their character, origins and relationship with the language of mathematics, the issue of Platonism
4. **Methodology and History:** Mathematical practice: its character, and the mathematical activities of mathematicians, in the present and past
5. **Applications and Values**: Applications of mathematics; its relationship with science, technology, other areas of knowledge and values
6. **Individual Knowledge and Learning**: The learning of mathematics: its character and role in the onward transmission of mathematical knowledge, and in the creativity of individual mathematicians (Ernest 1998)

Items 1 and 3 include the traditional epistemological and ontological focuses of the philosophy of mathematics, broadened to add a concern with the genesis of mathematical knowledge and objects of mathematics, as well as with language. Item 2 adds a concern with the form that mathematical knowledge usually takes: mathematical theories. Items 4 and 5 go beyond the traditional boundaries by admitting the applications of mathematics and human mathematical practice as legitimate philosophical concerns, as well as its relations with other areas of human knowledge and values. Item 6 adds a concern with how mathematics is transmitted onwards from one generation to the next, and in particular, how it is learnt by individuals, and the dialectical relation between individuals and existing knowledge in creativity.

The legitimacy of these extended concerns arises from the need to consider the relationship between mathematics and its corporeal agents, i.e., human beings. They are required to accommodate what on the face of it is the simple and clear task of the philosophy of mathematics, namely to give an account of mathematics.

3. Challenging Epistemological Assumptions and Values

The challenge to the traditional philosophy of mathematics to broaden its epistemological goal, as indicated above, raises some critical issues. In particular, if providing ironclad foundations to mathematical knowledge and mathematical truth is not the main purpose of philosophy of mathematics, has this fixation distorted philosophical accounts of mathematics and what is deemed valuable or significant in mathematics? To what extent is the philosophical emphasis on mathematical proof and deductive theories justified? I want to argue that the emphasis on mathematics as made up of rigorous deductive theories is excessive, and this focus in fact existed for only two periods totaling possibly less than ten percent of the overall history of mathematics as a systematic discipline, and then only in the West.[2]

The first of these two periods was the ancient Greek phase in the history of mathematics which reached its high point in the formulation of Euclid's *Elements*, a systematic exposition of deductive geometry and other topics. The second period is the modern era encompassing the past two hundred years or so. This second period was first signaled by Descartes' modernist epistemology, with its call to systematize all knowledge after the model of geometry in Euclid's *Elements*. However, fortunately, his injunction was not applied in the practices of mathematicians for the next two hundred years, which was instead a period of great creativity and invention in the West. Only in the 19th century did the newly professionalized mathematicians turn their attention to the foundations of mathematical knowledge and systematize it into axiomatic mathematical theories. The contributions of Boole, Weierstrass, Dedekind, Cantor, Peano, Hilbert, Frege, Russell and others in this enterprise up to the time of Bourbaki are well known.

I am not claiming that all or even most mathematical work was foundational during these two exceptional periods. But the foundational work is what caught the attention of philosophers of mathematics, and in the spirit of Cartesian modernism has become the epistemological focus of modern philosophy of mathematics, as well as the touchstone for what is deemed to be of epistemologically valuable. I do not want to detract from either the magnificence of the achievement in the foundational work carried out by mathematicians and logicians, nor from the pressing nature of the problems that made attention to it so vital in the early part of the 20th century. Nevertheless, the legacy of this attention has been to overvalue the philosophical significance of axiomatic mathematics at the expense of other dimensions of mathematics. Two underemphasized dimensions of mathematics are calculation and problem solving. All three of these aspects of mathematics involve deductive reasoning, but axiomatic mathematics is valued above the others as the supreme achievement of mathematics.

There is another feature shared by the two historical periods that emphasised axiomatic mathematics, namely a purist ideology involving the philosophical dismissal or rejection of the significance of practical mathematics. The antipathy of the ancient Greek philosophers to practical matters including numeration and calculation is well known. This aspect of mathematics was termed 'logistic' and regarded as the business of slaves or lesser beings. In the modern era, calculation and practical mathematics have been viewed as mathematically trivial and philosophically uninteresting. The fact that philosophers have been concerned with ontology and the nature of the mathematical objects has engendered little or no interest in the symbolism of mathematics, or calculations and transformations that convert one mathematical object (or rather its name, a term) into another. Such a view is typified by Platonism, which concerns itself primarily with mathematical truths and objects. These are presumed to exist in an unearthly and idealized world beyond that which we inhabit as fleshy and social human beings, such as Popper's (1979) objective World 3.

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[2] I take the beginning of disciplinary mathematics to be around 2500-3000 BCE, following Høyrup (1980) and (1994).

Of course at the same time as these modern developments were taking place applied mathematics and theoretical or mathematical physics were making great strides, but this was not considered to be of interest to philosophers of mathematics (however much interest it was to philosophers of science), because of their purist ideology. Even in British public schools, during the late Victorian era, mathematics was taught in with ungraduated rulers because graduations implied measurement and practical applications, which was looked down upon for the future professional classes and rulers of the country. (Admittedly some of the rationale was that Euclid's geometry only requires a straight-edge and a pair of compasses as drawing instruments).

What I have described here (in order to critique it) is an ideological perspective that elevates some aspects of mathematics above others, but typically does not acknowledge that it is based on a set of values, a set of choices and preferences to which no necessity or logical compulsion is attached. Furthermore, it appears that such values have only been prominent during a small part of the history of mathematics.

In order to strengthen my critique of these values I want to point out that mathematical proof, the cornerstone of axiomatic mathematics, and calculation in mathematics, are formally very close in structure and character. In Ernest (forthcoming) I have argued that mathematical topic areas (e.g., number and calculation) can be interpreted as being made up semiotic systems, each comprising (1) a set of signs, (2) rules of sign production and transformation, and (3) an underpinning (informal) meaning structure. Such signs include atomic, i.e., basic, signs and a range of composite signs comprising molecular constellations of atomic signs. These signs may be alphanumeric (made up of numerals or letters) or figural (e.g., geometric figures) or include both (e.g., figures with labels and the types of inference employed). The use of semiotic systems is primarily that of sign production in the pursuit of some goal (e.g., solving a problem, making a calculation, producing a proof for a theorem). I want to claim that most recorded mathematical activity concerns the production of sequences of signs (within a semiotic system). Typically these are transformations of an initial composite sign ($\boldsymbol{S_1}$), resulting after a finite number ($\boldsymbol{n}$) of transformations, in a terminal sign ($\boldsymbol{S_{n+1}}$), satisfying the requirements of the activity. This can be represented by the sequence: $\boldsymbol{S_1 \rightarrow S_2 \rightarrow S_3 \rightarrow \ldots \rightarrow S_{n+1}}$. Each transformation (represented by $\rightarrow$) constitutes the application of one of the rules of the semiotic system to the sign, resulting in the derivation of the next sign in the sequence. More accurately these should be represented by $\rightarrow_i$, with i = 1, …, n, since each transformation in the sequence is potentially different.

My claim is that this formal (semiotic) system describes most mathematical domains and activities. If the initial sign is the statement of a problem, the sequence represents the derivation of a solution to the problem. I will not dwell on this case as there are many complications involved in problem solving, such as the use of multiple representations, branching solution attempts[3], etc. and some of the transformations (such as interpreting an initial problem formulation and constructing a problem representation) are neither easily made explicit nor fully formalizable. Furthermore, there is no simple characterization of the relationship between the transformational rules and the underlying informal meaning structure, for the transformations are partly structure preserving morphisms, and partly calculational.

[3] Clearly branching derivations can occur in virtually all mathematical processes or activities including those involved in problem solving, deriving mathematical proofs, and mathematical calculations. However, they are mostly eliminated from the transcriptions of successfully completed activities.

More significantly in the present context, such transformational sequences can represent a deductive proof for a theorem. In this case it consists of a sequence of sentences, each of which is derived from its predecessors by the deductive rules of the system (including the introduction of axioms or other assumptions). The final sign in the sequence is the theorem proved. The meaning structure underpinning the rules of proof is based on the principle of the preservation of the truth value of sentences in each deductive step, and hence along the length of the proof sequence (which is why axioms can be inserted, and why proofs 'work', i.e., do what they are designed to do.)[4]

In the case of a calculation, the initial sign is usually a compound term. Subsequent terms are derived by calculational rules and typically each is a simplification in some sense of its predecessor. The final term in a calculation is the simplified numerical 'answer' to the problem. With the introduction of algebra and other functions and operations such as trigonometrical functions, the answer may instead be a simplified but non-numerical term (i.e., a function). Thus calculations are sequences of terms, each derived from predecessors by the rules of the system. The meaning structure underpinning the rules of calculation is based on the principle of the preservation of numerical value.[5]

Thus there is a strong analogy between the semiotic systems based on calculation and those based on deductive proof. The transformations of terms and sentences are based on the underlying principles of value preservation, namely numerical value or truth value, respectively, as I have demonstrated. In addition, calculation concerns terms and proof concerns sentences (or formulas), and both of these are defined similarly. Terms (sentences) are defined recursively as follows. An atomic term (sentence) consists of a constant or a variable (an n-place predicate applied to constants or variables, respectively). A compound term (sentence) is defined as the result of applying a function or operation (a logical connective or quantifier, respectively) to one or more terms (sentences, respectively), to make a new term (sentence, respectively). Thus structurally terms and sentences are very similar, defined analogously by induction.

The sequential and rule-based nature of calculation is something that precedes the development of the deductive proof of theorems by at least two thousand years. My contention is that without the long and ancient tradition of rule following in sequences of calculations, and the entrenchment of the grammatical and value preservational features noted above, the development of proof would not be possible. As I have indicated here, there is a striking analogy between calculations and deductive proofs of theorems, rarely if ever remarked upon, that puts into question the claimed superiority of proof.

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[4] Note that I have not distinguished between the two analogous forms of proof which employ equivalence or deductive consequence as the transformational relationship at each step. In the latter the truth value derived is greater than or equal to is precedent value, in the former it is equal to it. But in each case (in bivalent logic), since the initial truth value in the sequence must be 1 the whole sequence of truth values including the final term, the theorem proved, is 1.

[5] The preservation of one of the four inequality relations along the sequence is possible variation, where an upper or lower bound on the value of the term is determined

Furthermore, proof and calculation are formally equivalent, in modern foundational terms. Calculations utilize the term as a basic unit of meaning (and as that which is transformed), whereas deductive proofs use the sentence (including formulas or open sentences) as a basic unit. However, there are equivalence transformations between calculations and proofs. A calculation sequence of the form $t_1, t_2, t_3, \ldots, t_n$, where each t_i $(1 \leq i \leq n)$ is a term, can be represented as a deductive proof of the form $t_1=t_2, t_2=t_3, t_3=t_4, \ldots, t_{n-1}=t_n$ in which each identity asserts that numerical values of adjacent terms are preserved identically in the calculation. By an extended or repeated application of the transitivity of identity ($x=y \;\&\; y=z \rightarrow x=z$, for all x, y & z), $t_1=t_n$ is derived, thus equating the initial term of the calculation and the terminal term, the 'answer'.

Likewise, a deductive proof of the form $S_1, S_2, S_3, \ldots, S_n$, can be represented as a series of terms. These are the values of the truth value function f defined on numerical representations of true and false sentences to give the values 1 and 0, respectively. For a valid proof these values must be $f(S_1 \rightarrow S_2) = f(S_2 \rightarrow S_3) = \ldots = f(S_{n-1} \rightarrow S_n) = 1$. The formal details are messy and omitted here (see Gödel 1931 for the introduction of arithmetization of logic, and Kleene 1952) but the principle is both simple and sound. It is well known that f is a morphism mapping $<S, \rightarrow>$ onto a Boolean algebra $<f(S), \leq>$.[6]

The very strong analogy and structural similarities between proof and calculation, including their inter-convertibility, challenges the preconception often manifested in philosophical and historical accounts of mathematics that proof is somehow intellectually superior to calculation in mathematics.[7] Looking in detail at the technical and structural aspects of proof and calculation reveals that they cannot be so easily attributed to different epistemological domains as is often claimed. It is not defensible to say that proof alone in mathematics pertains to the true, good, beautiful, to wisdom, 'high-mindedness' and the transcendent dimensions of being, and that calculation is only technical and mechanical, pertaining to the utilitarian, practical, applied, and mundane; the lowly dimensions of existence. Such assertions are part of an ideological position incorporating a set of values that overvalues pure proof-based mathematics as having epistemological significance, and undervalues calculation and applied mathematics as having only practical significance; going back to the social divisions of ancient Greek society, as noted above, and the prejudices and ideology to which it gave rise.

This preconception or prejudice is used as the basis for asserting that the contributions of some cultures and civilizations are intellectually superior to others in history of mathematics. It also undervalues the solving of problems, calculations and other local applications of deduction in mathematics (including proof, see Joseph 1994). Thus the mathematics of ancient Egypt, Mesopotamia and India, as well as other countries outside of the Greco-European tradition, is viewed as inferior and immature. Part of the argument is that only cultures that produce axiomatic proof in mathematics achieve the highest levels of abstract intellectual achievement.

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[6] Technically the truth value function f can simply be defined on the domain of sentences under a given interpretation provided that there is an effective procedure for determining whether each sentence is true or false (thus giving values 1 or 0, respectively) under the given interpretation of the underlying theory or formal language.

[7] Joseph (1991) is among the few to note the importance of algorithms and calculation in the history of mathematics and to note their almost universal devaluation by other commentators.

I have argued that philosophical dispositions and values have underpinned a prejudice against ascribing value to certain forms of mathematical activity. In particular, that axiomatic systems are greatly valued over less systematic forms of deduction including problem solving, calculation and unsystematized proofs. Furthermore, this prejudice also maintains the contrast between and overvalues any form of proof, including unsystematized and unaxiomatized proofs over any form of calculation or problem solving.

These two levels of prejudice, these two value-based distinctions and preferences are frequently elided in the history and philosophy of mathematics. Thus the contributions of the ancient Greeks of the Euclidean type, and the modern focus on axiomatics of the past two centuries are seen to characterize the superior forms of thought of what is purported to be a Greco-European tradition. Furthermore, the unsystematized and unaxiomatized proofs and methods characterizing the official European history of mathematics from the late-Renaissance to the beginning on the Nineteenth century are seen as also reflecting the superior methods and concepts and higher forms of thought of the modern European tradition in their nascent phase, whose superiority and value is demonstrated in the subsequent flowering of the axiomatic tradition in Europe.

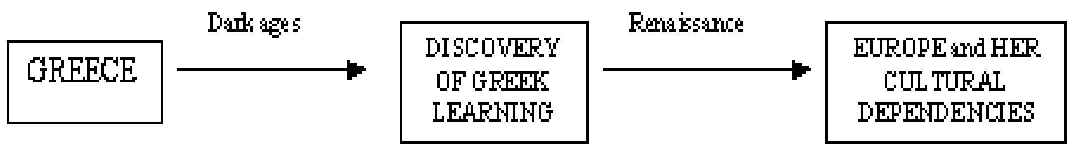
Through this elision, there arises the discounting of the proof-based contribution of cultures and civilizations outside of the 'Greco-European' tradition. Thus although there is a tradition of convincing demonstration or proofs, known as Upapatis, originating around two millennia ago in India, these proofs are discounted as intellectually inferior (Joseph 1994). Admittedly there are significant differences between the ancient Greek and the early Indian concepts of proof. Joseph 2000) has convincingly argued that ancient Indian mathematics was at least partially shaped by linguistic and grammatical conceptions of knowledge, based on the contributions of Panini; whereas ancient Greek mathematics was shaped by developments in philosophical thinking. So there are differences in the epistemological basis for different forms of proof that have emerged in different cultures and civilizations. However, the current challenges to the philosophy of mathematics discussed in the beginning of this paper, legitimate challenges to the traditional univocal and absolute conceptions of mathematics, knowledge and proof. From the perspective of the new fallibilist or social constructivist philosophies of mathematics, there is no ultimate or uniquely correct form of proof. Rather the forms of proof accepted within any culture or civilization during any epoch are a function of the historically contingent conceptual history and epistemological preconceptions that emerge and are accepted by the relevant geographico-historical communities of scholars. So there is no basis for elevating certain cultural forms of proof and demoting others on epistemological grounds alone. Each must be judged within the cultural contexts of its geographico-historical location.

4. Eurocentrism in the History and Philosophy of Mathematics

The above discussion raises the question of why informal and unsystematized proofs and demonstrations that occur in the mathematical histories of certain cultures are valued more than those of others. Why, for example, are the unsystematized proofs, methods and results of post-Renaissance European mathematics regarded as superior to antecedent developments in Kerala of comparable character? To answer this it is necessary to turn to another dimension of ideological prejudice at work in the history and philosophy of mathematics. This is eurocentrism, the racist bias that claims that the European 'mind' and its cultural products are superior to those of other peoples and races. Thus Bernal (1987) has argued that during the past two hundred year or so, ancient Greece has been 'talked up' as the starting point of modern European thought, and the 'Afroasiatic roots of Classical Civilisation' have been neglected, discarded and denied.

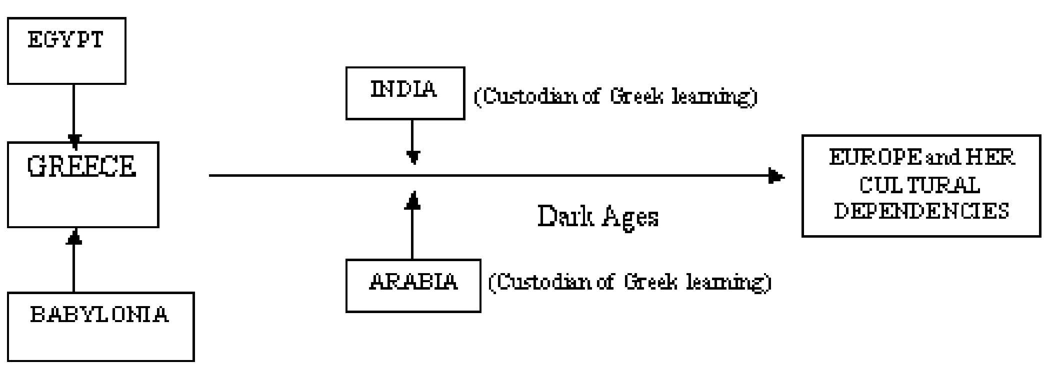
Against this backdrop it is not surprising that that mathematics has been seen as the product of European mathematicians. However, there is now a widespread literature supporting the thesis that mathematics has been misrepresented in a eurocentric way, including Almeida and Joseph (2004), Joseph (2000), Powell and Frankenstein (1997) and Pearce (undated). A common feature of eurocentric histories of mathematics is to claim that it was primarily the invention of the ancient Greeks. Their period ended almost 2000 years ago, which was followed by the 'dark ages' of around 1000 years until the European renaissance triggered by the rediscovery of Greek learning led to modern scientific and mathematical work in Europe (and its cultural dependencies). This trajectory is illustrated in Figure 1.

Figure 1: Eurocentric chronology of mathematics history (from Pearce, undated).



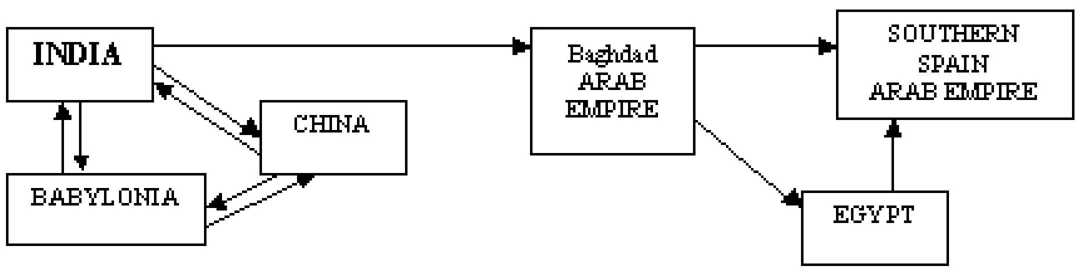
Some accounts have acknowledged the impact of lower level Egyptian and Babylonian mathematics on ancient Greek developments, as well as the later minor contributions of Indian and Arabic mathematicians (often seen primarily as custodians of Greek knowledge) on the history of mathematics in Europe (i.e., *The* history of mathematics). This is shown in the Modified Eurocentric model (figure 2).

Figure 2: Modified Eurocentric model (from Pearce, undated)



Pearce, Joseph and others go on to argue that in the so-called 'dark ages' and beyond, from 5th - 15th centuries, a great deal of mathematical work continued. Further the relationships between different regions and countries was complex and multidirectional and "A variety of mathematical activity and exchange between a number of cultural areas went on while Europe was in a deep slumber." (Joseph, 2000: 9). In figure 3 I reproduce Pearce's diagram of interrelationships in the development Non-European mathematics during the dark ages.

Figure 3: Non-European mathematics during the dark ages (from Pearce, undated)



Thus out of ignorance or prejudice arising from ideologically based values and preconceptions, eurocentric histories of mathematics, neglect the 'Non-European roots of mathematics' (to quote the subtitle of Joseph, 2000). There is a small but growing impact of such critical ideas in the history and philosophy of mathematics as indicated here. However, in my view, there is still an under-emphasis on the vital role of pre-Hellenic civilizations in providing the conceptual basis for modern mathematics through calculation, problem solving, etc.

5. Mathematics of the Indian Subcontinent and Kerala

One of the major casualties in the Eurocentric view of mathematics has been the ignoring or undervaluing of the contributions to mathematics of the Indian subcontinent. The long presence of deductive proofs in mathematics from this region has already been noted (Joseph 1994, 2000). Although the invention of zero by mathematicians of the Indian subcontinent has long been acknowledged, the significance of this as the lynchpin of the decimal place value system is often underestimated. Rotman (1987) presents a view of this innovation that puts its significance as reaching far beyond mathematics, right at the heart of European cultural and intellectual development in the Renaissance and early modern times. Pearce (undated) argues the Indian development of decimal numeration together with the place value system is the most remarkable development in the history of mathematics, as well as being one of the foremost intellectual productions in the overall history of humankind. I have indicated above how both philosophically and in the published histories of mathematics, calculation and numeration have traditionally been downplayed as epiphenomena of what is perceived to be the much more important Platonic conception of number. This is a misrepresentation of the intellectual significance of these developments without which the modern conceptions of number (including its computerization, with all of the applications this brings) would not be possible.

In the history of mathematics in the Indian subcontinent, much attention has been given to very large numbers, including powers of ten up to near 50. Whether these were contributors to or results of the development of the decimal place value system is for historians to say. Likewise it is tempting to speculate as to whether the extension of the decimal place value system into decimal fractions helped in the conceptualisation and formulation of the remarkable series expansions developed in Kerala. Although there is no unequivocal historical basis for this, it is convincingly claimed that floating point numbers were used by Kerala mathematicians to investigate the convergence of series (Almeida et al. 2001).

This brings me to one of the most remarkable and most neglected episodes in the history of mathematics, and the focus of this conference. This is the fact that Keralese mathematicians discovered and elaborated a large number of infinite series expansions and contributed much of the basis for the calculus, which is traditionally attributed to 17^{th} and 18^{th} century European mathematicians. Furthermore, this is not a case of simultaneous discovery in Kerala, for the work in Kerala took place two centuries before that in Europe.

Pearce (undated), Joseph (2000) and others attribute to Madhava of Sangamagramma (c. 1340 - 1425), the Keralese mathematician-astronomer, the important step of moving on from the finite procedures of ancient mathematics to treat their limit, the passage to infinity, the essence of modern classical analysis. He is also thought to have discovered numerous infinite series expansions of trigonometric and root terms, as well as for π, for which he calculated the value up to 13 (some say 17) decimal places (Pearce, undated). These inestimably important results anticipate some of the discoveries attributed to or named after the great mathematicians Gregory, Maclaurin, Taylor, Wallis, Newton, Leibniz and Euler.

Joseph (2000: 293) claims that "We may consider Madhava to have been the founder of mathematical analysis. Some of his discoveries in this field show him to have possessed extraordinary intuition". Almeida *et al*. (2001) have argued that Keralese contributions as a whole anticipate developments in Western Europe by several centuries in work on infinite series for numerical integration results.

In addition, these results are very possibly not just the anticipations of unacknowledged genius in the Indian subcontinent, and as such a very remarkable case of independent discovery. There is the very real possibility that these Keralese discoveries were transmitted to Europe by Jesuit missionaries and 'appropriated' by European mathematicians as their own (Almeida and Joseph 2004). The arguments for this transmission and appropriation are very persuasive, if not yet established with certainty. Certainly the mathematicians of Renaissance Europe are known to have been secretive about their methods and knowledge, and if they had 'purloined' the foundational results of calculus from Kerala would conceal and deny their origins.

As a non- historian of mathematics, I find this new recognition of the major Keralese and Indian subcontinent contributions to the history of mathematics remarkable. The fact that traditional histories of mathematics fail to acknowledge these and other non-European contributions is partly due to ignorance, for until recently it was difficult to find proper sources on this in standard texts. But there is much more to this, as there have been some reports of the anticipations in the literature for almost two centuries which have been disparaged or ignored.

Instead there are two sets of entwined ideological presuppositions that have led to this denial and blindness. The first is the epistemological prejudice towards a certain style of mathematics, namely the axiomatic theories and purist ideas discussed above as well as favoring proofs over calculation al and applied mathematics. Through the lenses of these modern prejudices the historical contributions of non-Eurocentric traditions has been minimized and trivialized. The second set of ideological presuppositions is more sinister. This is the racial prejudice of Eurocentrism. Namely, that only the 'Western mind' (i.e., the Caucasian or European) is capable of the pure thought and insights required in the highest forms of mathematics. Thus the contributions of African, Asian, Indian subcontinent, and Oriental peoples is discounted and minimized, because by the presupposed 'very inferior nature' of these peoples they are incapable of the high levels of thought involved. Hence any results that contradict these prejudices is *ab initio* incorrect. Thus such discoveries are minimized as intellectually inferior, or doubted and attributed to the transmission and copying of ideas from West to East, or in the last resort, challenged with regard to their chronology.

6. Conclusion

So what is the philosophical significance of the Keralese and Indian subcontinent contribution to history of mathematics? Identifying the most accurate genesis and trajectory of mathematical ideas in history that current knowledge allows should be the goal of every history of mathematics, and is consistent with any philosophy of mathematics. However, I have argued that a broader conceptualization of philosophy of mathematics is needed than the traditional emphasis on scholastic enquiries into epistemology and ontology. For such an emphasis has been associated, though I add need not necessarily be so, with an ideological position that devalues non-European contributions to history of mathematics. The philosophy of mathematics needs to be broad enough to recognise the salient features of the discipline it reflects upon, namely mathematics. As Lakatos (1976) indicated in the quote given above, the philosophy of mathematics has become empty by ignoring the history of mathematics.

It is no little charge to claim that the history and philosophy of mathematics have in effect become infused with error and a racist ideology, through the implicit and unacknowledged values and prejudices. Elsewhere, as well as above, I have argued that it is the business of mathematics and the philosophy of mathematics to take the issue of values seriously (Ernest 1998), and it is no longer enough to claim that these are outside of its proper subject matter. After all, ethics is just another branch of philosophy and I can see no grounds for its *a priori* exclusion. All human activities, however rarefied and abstruse are part of the vast cultural project of humankind, and as such none has the right to claim exemption from awareness of values and social responsibility (provided that this is not used as an excuse to limit freedom of thought and critique).

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